<u>3.1 Linear equation of Growth & Decay</u>(Differential Equations)

Questions:

1. The population of the community is known to increase at a rate proportional to the number of people present time t. if the population has doubled in 5 years, how long will it take to triple, to quadruple?
2. Suppose it is known that the population of the community in problem 1 is 10,000 after 3 years. What was the initial population in 10 years?
3. The population of a town grows at a rate proportional to the population present at time t. The initial population of 500 increases by 15 % in 10 years. What will be the population in 30 years?
4. Initially 100 ml of radioactive substance was present. After 6 hours, the mass has decreased by 3%. If the rate of decay is proportional to the amount of substance present at time t, find the amount remaining after 24 hours.
5. Determine the half life of the radioactive substance described in problem 4.

Solutions:

2. Suppose it is known that the population of the community in Problem 1 is 10,000 after 3 years. What was the initial population? What will be the population in 10 years?

$$P(3) = 10{,}000$$

$P_0 = ?,\ P(10) = ?$

$$Initial\ population\ function\ with\ respect\ to\ time\ P(t) = P_0 e^{kt}$$

$$\therefore P(3) = P_0 e^{k3} = P_0 e^{\frac{3}{5}ln2} \quad from\ problem\ 1$$

$$10000 = P_0 e^{ln2^{\frac{3}{5}}}$$

$$10000 = P_0 2^{\frac{3}{5}}$$

$$P_0 = \frac{10000}{2^{\frac{3}{5}}} = 6597.54$$

$$P(10) = P_0 e^{k.10} = 6597.54 e^{\frac{ln}{5}(10)} = 26390.2$$

9. Determine the half-life of the radioactive substance described in Problem 8.

$$r_0 = 100\ mg$$

$$\frac{dr}{dt} = rt$$

$$r(t) = r_0 e^{kt}$$

$$r(t) = 100e^{kt} - - - - - - - - -(1)$$

$$r(t) = \frac{1}{2}r_0 = \frac{1}{2}100 - - - - - - -(2) Condition\ of\ the\ problem$$

$$Compare\ (1) and\ (2)$$

$$100e^{kt} = \frac{1}{2}100$$

$$e^{kt} = \frac{1}{2}$$

$$e^{\frac{1}{6}\ln\left(\frac{97}{100}\right)t} = \frac{1}{2}$$

$$lne^{\frac{1}{6}\ln\left(\frac{97}{100}\right)t} = \ln\frac{1}{2}$$

$$\frac{1}{6}\ln\left(\frac{97}{100}\right)t = \ln\frac{1}{2}$$

$$\ln\left(\frac{97}{100}\right)t = 6ln\frac{1}{2} = -4.159$$

$$t = -\frac{4.159}{\ln\left(\frac{97}{100}\right)} = 136.5$$

$$Approximately\ 137\ years.$$